

High Temperature Confinement in $SU(N)$ Gauge Theories

Michael Ogilvie^{*†}

Washington University

E-mail: mco@physics.wustl.edu

Peter N. Meisinger

Washington University

E-mail: pnm@physics.wustl.edu

$SU(N)$ gauge theories, extended with adjoint fermions having periodic boundary conditions, are confining at high temperature for sufficiently light fermion mass m . Lattice simulations indicate that this confining region is smoothly connected to the confining region of low-temperature pure $SU(N)$ gauge theory. In the high temperature confining region, the one-loop effective potential for Polyakov loops has a $Z(N)$ -symmetric confining minimum. String tensions associated with Polyakov loops are smooth functions of m/T . In the magnetic sector, the Polyakov loop plays a role similar to a Higgs field, leading to a breaking of $SU(N)$ to $U(1)^{N-1}$. This in turn yields an effective theory where magnetic monopoles give rise to string tensions for spatial Wilson loops. These string tensions are calculable semiclassically. There are many analytical predictions for the high-temperature region that can be tested by lattice simulations, but lattice work will be crucial for exploring the crossover from this region to the low-temperature confining behavior of pure gauge theories.

The XXVI International Symposium on Lattice Field Theory

July 14 - 19, 2008

Williamsburg, Virginia, USA

^{*}Speaker.

[†]MCO is supported by a grant from the U.S. Dept. of Energy.

1. Introduction

One of the long-standing problems of modern strong-interaction physics is the origin of quark confinement. Finite temperature gauge theories are advantageous in many aspects for the study of confinement. The Polyakov loop operator P , given by

$$P(\vec{x}) = \mathcal{P} \exp \left[i \int_0^\beta dt A_4(\vec{x}, t) \right] \quad (1.1)$$

represents the insertion of a static quark into a thermal system of gauge fields. It is the order parameter for the deconfinement phase transition in pure $SU(N)$ gauge theories, with $\langle Tr_F P \rangle = 0$ in the confined phase, and $\langle Tr_F P \rangle \neq 0$ in the deconfined phase. The deconfinement phase transition is associated with the spontaneous breaking of a global $Z(N)$ symmetry $P \rightarrow zP$ where $z = \exp(2\pi i/N)$ is the generator of $Z(N)$.

It is remarkable that there is a simple class of $Z(N)$ -invariant systems that are confining at arbitrarily high temperatures, evading the transition to the deconfined phase found in the pure gauge theory. A high-temperature confined phase is obtained from a pure gauge theory by the addition of fermions in the adjoint representation of $SU(N)$, with the non-standard choice of periodic boundary conditions for the fermions in the timelike direction. If the number of adjoint fermion flavors N_f is not too large, these systems are asymptotically free at high temperature, and therefore the effective potential for P is calculable using perturbation theory. The system will lie in the confining phase if the fermion mass m is sufficiently light and $N_f > 1/2$. In this case, electric string tensions can be calculated perturbatively from the effective potential, and magnetic string tensions arise semiclassically from non-Abelian magnetic monopoles. Thus the high-temperature confining phase provides a realization of one of the oldest ideas about the origin of confinement.

2. High Temperature Confinement

The one-loop effective potential for a boson in a representation R with spin degeneracy s moving in a Polyakov loop background P at non-zero temperature and density is given by [1, 2]

$$V_b = sT \int \frac{d^d k}{(2\pi)^d} Tr_R \left[\ln \left(1 - P e^{\beta \mu - \beta \omega_k} \right) + \ln \left(1 - P^+ e^{-\beta \mu - \beta \omega_k} \right) \right]. \quad (2.1)$$

Periodic boundary conditions are assumed. With standard boundary conditions (periodic for bosons, antiperiodic for fermions), 1-loop effects always favor the deconfined phase. For the case of pure gauge theories, the one-loop effective potential can be written in the form

$$V_{gauge}(P, \beta, m, N_f) = \frac{-2}{\pi^2 \beta^4} \sum_{n=1}^{\infty} \frac{Tr_A P^n}{n^4}. \quad (2.2)$$

This series is minimized, term by term if $P \in Z(N)$, so $Z(N)$ symmetry is spontaneously broken at high temperature. The same result is obtained for any bosonic field with periodic boundary conditions or for fermions with antiperiodic boundary conditions.

The addition of fermions with periodic boundary conditions can restore the broken $Z(N)$ symmetry. Consider the case of N_f flavors of Dirac fermions in the adjoint representation of $SU(N)$.

Periodic boundary conditions in the timelike direction imply that the generating function of the ensemble, *i.e.*, the partition function, is given by

$$Z = \text{Tr} \left[(-1)^F e^{-\beta H} \right] \quad (2.3)$$

where F is the fermion number. This ensemble, familiar from supersymmetry, can be obtained from an ensemble at chemical potential μ by the replacement $\beta\mu \rightarrow i\pi$. In perturbation theory, this shifts the Matsubara frequencies from $\beta\omega_n = (2n+1)\pi$ to $\beta\omega_n = 2n\pi$. The one loop effective potential is like that of a bosonic field, but with an overall negative sign due to fermi statistics [3]. The sum of the effective potential for the fermions plus that of the gauge bosons gives

$$V_{1-loop}(P, \beta, m, N_f) = \frac{1}{\pi^2 \beta^4} \sum_{n=1}^{\infty} \frac{\text{Tr}_A P^n}{n^2} \left[2N_f \beta^2 m^2 K_2(n\beta m) - \frac{2}{n^2} \right]. \quad (2.4)$$

Note that the first term in brackets, due to the fermions, is positive for every value of n , while the second term, due to the gauge bosons, is negative.

The largest contribution to the effective potential at high temperatures is typically from the $n = 1$ term, which can be written simply as

$$\frac{1}{\pi^2 \beta^4} \left[2N_f \beta^2 m^2 K_2(\beta m) - 2 \right] \left[|\text{Tr}_F P|^2 - 1 \right] \quad (2.5)$$

where the overall sign depends only on N_f and βm . If $N_f \geq 1$ and βm is sufficiently small, this term will favor $\text{Tr}_F P = 0$. On the other hand, if βm is sufficiently large, a value of P from the center, $Z(N)$, is preferred. Note that an $\mathcal{N} = 1$ super Yang-Mills theory would correspond to $N_f = 1/2$ and $m = 0$, giving a vanishing perturbative contribution for all n [4, 5]. This suggests that it should be possible to obtain a $Z(N)$ symmetric, confining phase at high temperatures using adjoint fermions with periodic boundary conditions or some equivalent deformation of the theory.

This possibility has been confirmed in $SU(3)$, where both lattice simulations and perturbative calculations have been used to show that a gauge theory action with an extra term of the form $\int d^4x a_1 \text{Tr}_A P$ is confining for sufficiently large a_1 at arbitrarily high temperatures [6]. This simple, one-term deformation is sufficient for $SU(2)$ and $SU(3)$. However, in the general case, a deformation with at least $\lceil \frac{N}{2} \rceil$ terms is needed to assure confinement for representations of all possible non-zero k -alities. Thus the minimal deformation necessary is of the form

$$\sum_{k=1}^{\lceil \frac{N}{2} \rceil} a_k \text{Tr}_A P^k \quad (2.6)$$

which is analyzed in detail in Joyce Myer's presentation in these proceedings [7]. If all the coefficients a_k are sufficiently large and positive, the free energy density

$$V_{1-loop}(P, \beta, m, N_f) = \frac{-2}{\pi^2 \beta^4} \sum_{n=1}^{\infty} \frac{\text{Tr}_A P^n}{n^4} + \sum_{k=1}^{\lceil \frac{N}{2} \rceil} a_k \text{Tr}_A P^k \quad (2.7)$$

will be minimized by a unique set of Polyakov loop eigenvalues corresponding to exact $Z(N)$ symmetry.

The unique set of $SU(N)$ Polyakov eigenvalues invariant under $Z(N)$ is $\{w, wz, wz^2, \dots, wz^{N-1}\}$, where $z = e^{2\pi i/N}$ is the generator of $Z(N)$, and w is a phase necessary to ensure unitarity [8]. A matrix with these eigenvalues, such as $P_0 = w \cdot \text{diag}[1, z, z^2, \dots, z^{N-1}]$, is gauge-equivalent to itself after a $Z(N)$ symmetry operation: $zP_0 = gP_0g^\dagger$. This guarantees that $\text{Tr}_F[P_0^k] = 0$ for any value of k not divisible by N , indicating confinement for all representations transforming non-trivially under $Z(N)$.

To prove that P_0 is a global minimum of the effective potential, we use the high-temperature expansion for the one-loop free energy of a particle in an arbitrary background Polyakov loop gauge equivalent to the matrix $P_{jk} = \delta_{jk}e^{i\phi_j}$. The first two terms have the form [3]

$$V_{1-loop} \approx \sum_{j,k=1}^N \left(1 - \frac{1}{N}\delta_{jk}\right) \frac{2(2N_f - 1)T^4}{\pi^2} \left[\frac{\pi^4}{90} - \frac{1}{48\pi^2} (\phi_j - \phi_k)^2 (\phi_j - \phi_k - 2\pi)^2 \right] \\ - \sum_{j,k=1}^N \left(1 - \frac{1}{N}\delta_{jk}\right) \frac{N_f m^2 T^2}{\pi^2} \left[\frac{\pi^2}{6} + \frac{1}{4} (\phi_j - \phi_k) (\phi_j - \phi_k - 2\pi) \right]. \quad (2.8)$$

The T^4 term dominates for $m/T \ll 1$, and has P_0 as a minimum provided $N_f > 1/2$. Even if the adjoint fermion mass is enhanced by chiral symmetry breaking, as would be expected in a confining phase, it should be of order gT or less, and the second term in the expansion of V_{1-loop} can be neglected at sufficiently high temperature.

3. Temporal String Tensions

The timelike string tension $\sigma_k^{(t)}$ between k quarks and k antiquarks can be measured from the behavior of the correlation function

$$\left\langle \text{Tr}_F P^k(\vec{x}) \text{Tr}_F P^{+k}(\vec{y}) \right\rangle \simeq \exp \left[-\frac{\sigma_k^{(t)}}{T} |\vec{x} - \vec{y}| \right] \quad (3.1)$$

at sufficiently large distances. Two widely-considered scaling behaviors for string tensions are Casimir scaling, characterized by

$$\sigma_k = \sigma_1 \frac{k(N-k)}{N-1}, \quad (3.2)$$

$$(3.3)$$

and sine-law scaling, given by

$$\sigma_k = \sigma_1 \sin \left[\frac{\pi k}{N} \right]. \quad (3.4)$$

For a review, see reference [9]. Timelike string tensions are calculable perturbatively in the high-temperature confining region from small fluctuations about the confining minimum of the effective potential [10]. The scale is naturally of order gT :

$$\left(\frac{\sigma_k^{(t)}}{T} \right)^2 = g^2 N \frac{2N_f m^2}{2\pi^2} \sum_{j=0}^{\infty} [K_2((k+jN)\beta m) + K_2((N-k+jN)\beta m) - 2K_2((j+1)N\beta m)] \\ - g^2 N \frac{T^2}{3N^2} \left[3 \csc^2 \left(\frac{\pi k}{N} \right) - 1 \right]. \quad (3.5)$$

These string tensions are continuous functions of βm . The $m = 0$ limit is simple:

$$\left(\frac{\sigma_k^{(t)}}{T}\right)^2 = \frac{(2N_f - 1)g^2 T^2}{3N} \left[3 \csc^2\left(\frac{\pi k}{N}\right) - 1 \right] \quad (3.6)$$

and is a good approximation for $\beta m \ll 1$. This scaling law is not at all like either Casimir or sine-law scaling, because the usual hierarchy $\sigma_{k+1}^{(t)} \geq \sigma_k^{(t)}$ is here reversed. Because we expect on the basis of $SU(3)$ simulations that the high-temperature confining region is continuously connected to the conventional low-temperature region, there must be an inversion of the string tension hierarchy between the two regions for all $N \geq 4$.

4. Spatial String Tensions

The confining minimum P_0 of the effective potential breaks $SU(N)$ to $U(1)^{N-1}$. This remaining unbroken Abelian gauge group naively seems to preclude spatial confinement, in the sense of area law behavior for spatial Wilson loops. However, as first discussed by Polyakov in the case of an $SU(2)$ Higgs model in $2 + 1$ dimensions, instantons can lead to nonperturbative confinement [11]. In the high-temperature confining region, the dynamics of the magnetic sector are effectively three-dimensional due to dimensional reduction. The Polyakov loop plays a role similar to an adjoint Higgs field, with the important difference that P lies in the gauge group, while a Higgs field would lie in the gauge algebra. The standard topological analysis [12] is therefore slightly altered, and there are N fundamental monopoles in the finite temperature gauge theory [13, 14, 15, 16, 17] with charges proportional to the affine roots of $SU(N)$, given by $2\pi\alpha_j/g$ where $\alpha_j = \hat{e}_j - \hat{e}_{j+1}$ for $j = 1$ to $N - 1$ and $\alpha_N = \hat{e}_N - \hat{e}_1$. Monopole effects will be suppressed by powers of the Boltzmann factor $\exp[-E_j/T]$ where E_j is the energy of a monopole associated with α_j .

In the high-temperature confining region, monopoles interact with each other through both their long-ranged magnetic fields, and also via a three-dimensional scalar interaction, mediated by A_4 . The scalar interaction is short-ranged, falling off with a mass of order gT . The long-range properties of the magnetic sector may be represented in a simple form by a generalized sine-Gordon model which generates the grand canonical ensemble for the monopole/anti-monopole gas [18]. The action for this model represents the Abelian dual form of the magnetic sector of the $U(1)^{N-1}$ gauge theory. It is given by

$$S_{mag} = \int d^3x \left[\frac{T}{2} (\partial\rho)^2 - 2\xi \sum_{j=1}^N \cos\left(\frac{2\pi}{g}\alpha_j \cdot \rho\right) \right] \quad (4.1)$$

where ρ is the scalar field dual to the $U(1)^{N-1}$ magnetic field. The monopole fugacity ξ is given by $\exp[-E_j/T]$ times functional determinantal factors [19].

This Lagrangian is a generalization of the one considered by Polyakov for $SU(2)$, and the analysis of magnetic confinement follows along the same lines [11]. The Lagrangian has N degenerate inequivalent minima $\rho_{0k} = g\mu_k$ where the μ_k 's are the simple fundamental weights, satisfying $\alpha_j \cdot \mu_k = \delta_{jk}$. Note that $e^{2\pi i \mu_k} = z^k$. A spatial Wilson loop

$$W[\mathcal{C}] = \mathcal{P} \exp \left[i \oint_{\mathcal{C}} dx_j \cdot A_j \right] \quad (4.2)$$

in the x - y plane introduces a discontinuity in the z direction in the field dual to B . Moving this discontinuity out to spatial infinity, the string tension of the spatial Wilson loop is the interfacial energy of a one-dimensional kink interpolating between the vacua ρ_{0k} . The calculation is similar to that of the 't Hooft loop in the deconfined phase, where the kinks interpolate between the N different solutions associated with the spontaneous breaking of $SU(N)$. The main technical difficulty lies in finding the correct kink solutions. A straight line ansatz through the Lie algebra [20] using

$$\rho(z) = g\mu_k q(z) \quad (4.3)$$

gives

$$\sigma_k^{(s)} = \frac{8}{\pi} \left[\frac{g^2 T \xi}{N} k(N-k) \right]^{1/2} \quad (4.4)$$

This result is exact for $N = 2$ or 3 , but may be only an upper bound for $N > 3$. The square-root-Casimir scaling behavior obtained differs significantly from both Casimir and sine-law scaling, and should be easily distinguishable in lattice simulations.

5. Conclusions

We have been able to predict analytically a number of properties of the high temperature confined region, which lattice simulations should be able to confirm. The phase structure and thermodynamics of these models are particularly rich for $N \geq 4$. However, even in the case of $SU(2)$, there is an interesting prediction of a first-order transition between the deconfined phase and the high-temperature confined region as the adjoint fermion mass is varied. It follows that there must be a tricritical point in the $\beta - m$ plane somewhere on the critical line separating the confined and deconfined phases. There is a perturbative prediction for an inverted hierarchy of timelike string tensions for $N \geq 4$. There is also a semiclassical expression for spacelike string tensions, which are predicted to be proportional to the square root of the monopole fugacity.

These predictions for the high-temperature region lead naturally to additional questions that lattice simulations can address, but semiclassical methods most likely cannot. Lattice simulations can explore the crossover from conventional, low-temperature confining behavior to the behavior predicted in the high-temperature confining region. Some features can be studied in simulations where a simple deformation of the action is used, as in [6]. In addition to the string tensions, these features include monopole and instanton densities, and the topological susceptibility. Other aspects will require the inclusion of adjoint dynamical fermions in lattice simulations. Chiral symmetry breaking is of particular interest. Unsal [21, 22] has proposed a detailed picture of chiral symmetry breaking which can be independently checked by simulation. The accessibility of lattice field configurations as well as conventional observables makes the high-temperature confined region a natural place to explore the overlap of theory and simulation.

References

- [1] D. J. Gross, R. D. Pisarski and L. G. Yaffe, *QCD And Instantons At Finite Temperature*, Rev. Mod. Phys. **53** (1981) 43.

- [2] N. Weiss, *The Effective Potential For The Order Parameter Of Gauge Theories At Finite Temperature*, Phys. Rev. D **24** (1981) 475.
- [3] P. N. Meisinger and M. C. Ogilvie, *Complete high temperature expansions for one-loop finite temperature effects*, Phys. Rev. D **65** (2002) 056013 [arXiv:hep-ph/0108026].
- [4] N. M. Davies, T. J. Hollowood, V. V. Khoze and M. P. Mattis, *Gluino condensate and magnetic monopoles in supersymmetric gluodynamics*, Nucl. Phys. B **559** (1999) 123 [arXiv:hep-th/9905015].
- [5] N. M. Davies, T. J. Hollowood and V. V. Khoze, *Monopoles, affine algebras and the gluino condensate*, J. Math. Phys. **44** (2003) 3640 [arXiv:hep-th/0006011].
- [6] J. C. Myers and M. C. Ogilvie, *New Phases of $SU(3)$ and $SU(4)$ at Finite Temperature*, Phys. Rev. D **77** (2008) 125030 [arXiv:0707.1869 [hep-lat]].
- [7] J. C. Myers and M. C. Ogilvie, *Exotic phases of finite temperature $SU(N)$ gauge theories with massive fermions: F , Adj , A/S* , arXiv:0809.3964 [hep-lat].
- [8] P. N. Meisinger, T. R. Miller and M. C. Ogilvie, *Phenomenological equations of state for the quark-gluon plasma*, Phys. Rev. D **65** (2002) 034009 [arXiv:hep-ph/0108009].
- [9] J. Greensite, *The confinement problem in lattice gauge theory*, Prog. Part. Nucl. Phys. **51** (2003) 1 [arXiv:hep-lat/0301023].
- [10] P. N. Meisinger and M. C. Ogilvie, *Polyakov loops, $Z(N)$ symmetry, and sine-law scaling*, Nucl. Phys. Proc. Suppl. **140** (2005) 650 [arXiv:hep-lat/0409136].
- [11] A. M. Polyakov, *Quark Confinement And Topology Of Gauge Groups*, Nucl. Phys. B **120** (1977) 429.
- [12] E. J. Weinberg, *Fundamental Monopoles And Multi-Monopole Solutions For Arbitrary Simple Gauge Groups*, Nucl. Phys. B **167** (1980) 500.
- [13] K. M. Lee, *Instantons and magnetic monopoles on $R^{*3} \times S(1)$ with arbitrary simple gauge groups*, Phys. Lett. B **426** (1998) 323 [arXiv:hep-th/9802012].
- [14] T. C. Kraan and P. van Baal, *Exact T-duality between calorons and Taub - NUT spaces*, Phys. Lett. B **428** (1998) 268 [arXiv:hep-th/9802049].
- [15] K. M. Lee and C. h. Lu, *$SU(2)$ calorons and magnetic monopoles*, Phys. Rev. D **58** (1998) 025011 [arXiv:hep-th/9802108].
- [16] T. C. Kraan and P. van Baal, *Periodic instantons with non-trivial holonomy*, Nucl. Phys. B **533** (1998) 627 [arXiv:hep-th/9805168].
- [17] T. C. Kraan and P. van Baal, *Monopole constituents inside $SU(n)$ calorons*, Phys. Lett. B **435** (1998) 389 [arXiv:hep-th/9806034].
- [18] M. Unsal and L. G. Yaffe, *Center-stabilized Yang-Mills theory: confinement and large N volume independence*, arXiv:0803.0344 [hep-th].
- [19] K. Zarembo, *Monopole determinant in Yang–Mills theory at finite temperature*, Nucl. Phys. B **463** (1996) 73 [arXiv:hep-th/9510031].
- [20] P. Giovannangeli and C. P. Korthals Altes, *'t Hooft and Wilson loop ratios in the QCD plasma*, Nucl. Phys. B **608** (2001) 203 [arXiv:hep-ph/0102022].
- [21] M. Unsal, *Abelian duality, confinement, and chiral symmetry breaking in QCD(adj)*, Phys. Rev. Lett. **100** (2008) 032005 [arXiv:0708.1772 [hep-th]].
- [22] M. Unsal, *Magnetic bion condensation: A new mechanism of confinement and mass gap in four dimensions*, arXiv:0709.3269 [hep-th].